

Spin polarization decay in spin-1/2 and spin-3/2 systems

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We present a general unifying theory for spin polarization decay due to the interplay of spin precession and momentum scattering that is applicable to both spin-1/2 electrons and spin-3/2 holes. Our theory allows us to identify and characterize a wide range of qualitatively different regimes. For strong momentum scattering or slow spin precession we recover the D'yakonov-Perel result, according to which the spin relaxation time is inversely proportional to the momentum relaxation time. On the other hand, we find that, in the ballistic regime the carrier spin polarization shows a very different qualitative behavior. In systems with isotropic spin splitting the spin polarization can oscillate indefinitely, while in systems with anisotropic spin splitting the spin polarization is reduced by spin dephasing, which is non-exponential and may result in an incomplete decay of the spin polarization. For weak momentum scattering or fast spin precession, the oscillations or non-exponential spin dephasing are modulated by an exponential envelope proportional to the momentum relaxation time. Nevertheless, even in this case in certain systems a fraction of the spin polarization may survive at long times. Finally it is shown that, despite the qualitatively different nature of spin precession in the valence band, spin polarization decay in spin-3/2 hole systems has many similarities to its counterpart in spin-1/2 electron systems.

I. INTRODUCTION

The achievement of a lasting spin polarization has been a long-standing goal in semiconductor physics. Successful efforts to *generate* a spin polarization magnetically, optically and electrically, have yielded a steady stream of novel physics and promising applications.^{1,2} Ferromagnetic semiconductors are edging towards room temperature³ and spin currents have been measured directly.⁴ Successes such as these have turned semiconductor spin electronics into a vibrant and rewarding area of research, as well as a promising candidate for novel information processing methods.

Both for fundamental physics and for technological applications, it is important to know how to *maintain* a spin polarization once it is generated. Therefore, a detailed understanding of the mechanisms leading to spin polarization decay is critical in all areas mentioned above. In the return to equilibrium of an excess spin polarization spin-orbit interactions play an important role. Spin-orbit coupling always gives rise to spin precession, and the interplay of spin precession and momentum scattering is frequently the main cause of spin polarization decay.^{5,6,7} A spin polarization in a semiconductor may also decay via spin flips induced by momentum scattering or by exchange interactions, though these mechanisms have a more limited range of applicability.^{7,8,9,10}

Spin relaxation in spin-1/2 electron systems has received considerable attention.^{5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28} For electrons the spin-orbit interaction can always be represented by a Zeeman-like Hamiltonian $H = (\hbar/2) \boldsymbol{\sigma} \cdot \boldsymbol{\Omega}(\mathbf{k})$ describing the interaction of the spin with an effective wave vector-dependent magnetic field $\boldsymbol{\Omega}(\mathbf{k})$. The electron spin precesses about this field with frequency $\Omega \equiv |\boldsymbol{\Omega}(\mathbf{k})|$. An important parameter is

the product of the frequency Ω times the momentum relaxation time τ_p . In the ballistic (clean) regime no scattering occurs and the temperature tends to absolute zero, so that $\Omega\tau_p \rightarrow \infty$. The weak scattering regime is characterized by fast spin precession and little momentum scattering due to, e.g., a slight increase in temperature, yielding $\Omega\tau_p \gg 1$. In the strong momentum scattering regime $\Omega\tau_p \ll 1$.

Electron systems are often in the strong scattering regime. In this case the main mechanism leading to spin polarization decay is the D'yakonov-Perel (DP) mechanism,^{5,6,7} which was shown to be dominant over a wide range of temperatures⁷ and, for particular forms of $\boldsymbol{\Omega}(\mathbf{k})$, to lead to a noticeable anisotropy in the relaxation times for different spin components^{13,14} and anisotropic spin diffusion.¹⁹ Most past work has concentrated on this regime. On the other hand, in recent years state-of-the-art technology has enabled the growth of ballistic samples which have been at the forefront of spin-related experiments.^{1,2} Yet spin polarization decay in ballistic spin-1/2 systems has received comparably little attention^{1,20,21} and has been considered recently mostly in the context of spin transport in an electric field.^{17,18}

For spin-3/2 holes the spin-orbit interaction cannot be written as an effective field, and spin precession is qualitatively different.²⁹ Since spin-orbit coupling is more important in the valence band, hole spin information is lost faster, and the relative strengths of spin-orbit coupling and momentum scattering can vary. Yet spin relaxation of spin-3/2 holes has also been studied to a lesser extent, both experimentally³⁰ and theoretically.^{8,31,32,33,34,35} A theory of spin relaxation valid for electrons and holes in all regimes of momentum scattering does not, to our knowledge, exist to date.

With these observations in mind, we present in this article a general unifying quantitative theory for the re-

turn to equilibrium of excess spin polarizations in the conduction and valence bands of semiconductors brought about by the interplay of spin precession and momentum scattering. We do not rely on the assumption, made in most previous work,^{5,6,7,8,11,12,13,14,15,16,17} that $\Omega \tau_p \ll 1$. We demonstrate that spin polarization decay in different regimes of momentum scattering in spin-1/2 electron and spin-3/2 hole systems contains considerable rich and novel physics. For example, spin polarization decay has often been assumed to be proportional to e^{-t/τ_s} , where τ_s is referred to as the *spin relaxation time*. However, if the magnitude of the spin-orbit interaction is anisotropic (as is usually the case in systems studied experimentally), spin-polarization decay can occur even in the absence of momentum scattering. This process is characterized by a non-exponential decay and is sensitive to the initial conditions, and cannot therefore be described by a spin relaxation time. Weak momentum scattering introduces a spin relaxation time $\tau_s \propto \tau_p$ (unlike strong momentum scattering, which gives the well-known^{5,7} trend $\tau_s \propto \tau_p^{-1}$), yet even in the presence of weak momentum scattering a fraction of the polarization may survive at long times. It will emerge from our work that, in the ballistic and weak momentum scattering regimes, the concept of a spin relaxation time is of very limited applicability and in general does not provide an accurate description of the physics of spin polarization decay.

We emphasize that the results presented in this paper are true for (delocalized) electron spins in *any* nonmagnetic solid where spin-orbit coupling is important. Since in today's experiments mobilities range over many orders of magnitude, the results presented are directly relevant to ongoing state-of-the-art research.

The outline of this article is as follows. In section II we discuss the time evolution of the density matrix, deriving an equation which describes the return to equilibrium of a spin polarization. We demonstrate that in the general case there exists a fraction of the spin polarization which does not precess, and explain its relevance to the subsequent time evolution of the spin polarization. Section III is devoted to spin-1/2 electron systems, in which first the known D'yakonov-Perel' limit is discussed, then the complex situations in the ballistic and weak momentum scattering regimes are presented. We stress the importance of non-exponential decay and of incomplete spin dephasing. Finally, in the last part we demonstrate that, although spin precession is qualitatively different in spin-1/2 electron and spin-3/2 hole systems, spin polarization decay in these systems can be understood based on the same fundamental concepts.

II. TIME EVOLUTION OF THE DENSITY MATRIX

We assume a nonequilibrium spin polarization has been generated in a homogeneous, unstructured system and study its time evolution in the absence of exter-

nal fields. The system is described by a density matrix, which in principle has matrix elements diagonal and off-diagonal in momentum space. Since the spin operator is diagonal in the wave vector \mathbf{k} , we will only be concerned with the part of the density matrix diagonal in momentum space, which is denoted by ρ . Henceforth, by "density matrix" we understand the part of the density matrix diagonal in wave vector.

The spin density is given by $\langle \mathbf{S} \rangle \equiv \text{tr} \mathbf{S} \rho = \text{tr} \mathbf{S} \bar{\rho}$, where \mathbf{S} is the spin operator, and the overline represents averaging over directions in momentum space. Only the isotropic part $\bar{\rho}$ of the density matrix is responsible for spin population decay.⁷ It is therefore convenient to divide ρ into $\rho = \bar{\rho} + g$, where g is the anisotropic part of ρ . Based on the quantum Liouville equation, we obtain an equation describing the time evolution of ρ (Ref. 8), which in turn is split into a set of equations for $\bar{\rho}$ and g similar to those found by Pikus and Titkov.⁷

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{i}{\hbar} [\overline{H, g}] = 0, \quad (1a)$$

$$\frac{\partial g}{\partial t} + \frac{i}{\hbar} [H, g] + \frac{g}{\tau_p} = -\frac{\partial \bar{\rho}}{\partial t} - \frac{i}{\hbar} [H, \bar{\rho}]. \quad (1b)$$

These equations hold both for spin-1/2 electrons and for spin-3/2 holes. We assume elastic scattering by short-range impurities, implying that the collision term involving $\bar{\rho}$ vanishes⁷ and the remainder is proportional to the inverse of the scalar momentum relaxation time³⁶ $1/\tau_p$.

Before proceeding, we would like to make two remarks concerning the form of the scattering term. Firstly, in the presence of spin-orbit coupling both intraband and interband transitions exist, while we have assumed a simplified form of the scattering term. In the version of the relaxation time approximation employed in this work the spin splitting of the bands is not taken into account in the scattering term. This approximation is justified by the fact that spin eigenstates are generally *not* energy eigenstates, and it can be straightforwardly shown, based on the theory we present, that accounting explicitly for interband transitions will not change the fundamental physics of spin polarization decay, rather it will only give less transparent solutions. Furthermore, spin-flip scattering in nonmagnetic systems is third-order in the scattering potential and/or first order in the ratio of the spin-orbit splitting and the kinetic energy.

Secondly, it should be noted that, for degenerate carriers, the return to equilibrium requires energy dissipation. However, as noted above, in a nonmagnetic material with spin-orbit coupling the spin eigenstates characterizing the nonzero spin polarization are not energy eigenstates. On the other hand, unlike in, e.g., nuclear systems, the nonthermal energy characterizing this nonequilibrium configuration is essentially a kinetic energy, but it is not in the spin degree of freedom. Therefore, energy dissipation has no qualitative effect for the main conclusions in our paper.

A solution to Eq. (1b) can be obtained by making the transformation $g = e^{-iHt/\hbar} g_I e^{iHt/\hbar}$, which is analogous

to the customary switch to the interaction picture. This transformation turns Eq. (1b) into an equation for g_I

$$\frac{\partial g_I}{\partial t} + \frac{g_I}{\tau_p} = -\frac{\partial \bar{\rho}_I}{\partial t}, \quad (2)$$

where $\bar{\rho}_I$ is defined by $\bar{\rho} = e^{-iHt/\hbar} \bar{\rho}_I e^{iHt/\hbar}$. Treating the RHS as a source term, this equation allows an analytical solution using an integrating factor. Substituting this solution into Eq. (1a) yields

$$\begin{aligned} \frac{\partial \bar{\rho}}{\partial t} + \frac{i}{\hbar \tau_p} \int_0^t dt' e^{-(t-t')/\tau_p} \overline{e^{-iH(t-t')/\hbar} [H, \bar{\rho}(t')] e^{iH(t-t')/\hbar}} \\ = -\frac{i}{\hbar} e^{-t/\tau_p} \overline{e^{-iHt/\hbar} [H, \rho_0] e^{iHt/\hbar}}, \end{aligned} \quad (3)$$

where ρ_0 is the initial value $\rho(t=0)$. This equation is the main result of our paper. It describes the precession-induced decay of spin polarization in all regimes of momentum scattering for any nonmagnetic solid state system with spin-orbit interactions. This equation does not anticipate any particular form of spin polarization decay, such as exponential decay.

The form of the initial density matrix ρ_0 is important and lies at the root of the novel physics discussed in this paper. In general ρ_0 has two contributions, $\rho_0 = \rho_{0\parallel} + \rho_{0\perp}$. The component $\rho_{0\parallel}$ commutes with H and is given by $\rho_{0\parallel} = (\text{tr } \rho_0 H / \text{tr } H^2) H$, in a generalization of Gram-Schmidt orthogonalization. $\rho_{0\perp}$ is simply the remainder, and it satisfies the condition $\text{tr } \rho_{0\perp} H = 0$. $\rho_{0\parallel}$ is a matrix that is parallel to the Hamiltonian, and represents the fraction of the initial spin polarization that does not precess, or alternatively the fraction of the initial spins that are in eigenstates of the Hamiltonian. $\rho_{0\perp}$ is orthogonal to the Hamiltonian, and represents the fraction of the initial spin polarization that does precess.

III. SPIN-1/2 ELECTRON SYSTEMS

First we discuss Eq. (3) for spin-1/2 systems. The Hamiltonian describing spin-orbit coupling has the form $H = (\hbar/2) \boldsymbol{\sigma} \cdot \boldsymbol{\Omega}(\mathbf{k})$ and $\bar{\rho}$ may be decomposed as $\bar{\rho} = \frac{1}{2} [n + \mathbf{s}(t) \cdot \boldsymbol{\sigma}]$, where n represents the number density and $\mathbf{s}(t)$ the spin polarization. Equation (3) has qualitatively different solutions depending on the regime under study, and they are discussed in detail below.

A. Exponential decay in the strong momentum scattering regime

A solution to Eq. (3) characterizing *relaxation* is understood as *exponential* decay of the form $\bar{\rho}(t) = e^{-\Gamma_s t} \bar{\rho}_0$, where Γ_s is generally a second-rank tensor that represents the inverse of the spin relaxation time τ_s . Such a simple solution of Eq. (3) does *not* exist in general, but for strong momentum scattering ($\Omega \tau_p \ll 1$) the RHS of Eq. (3) can

be neglected. Then substituting for $\bar{\rho}$ and H in Eq. (3) yields the DP expression^{5,7} for Γ_s , which may be written as $(\Gamma_s)_{ij} = \tau_p (\overline{\Omega^2 \delta_{ij} - \Omega_i \Omega_j})$, where $i, j = x, y, z$. Strong momentum scattering yields exponential spin relaxation and the well-known^{5,7} trend $\tau_s \propto \tau_p^{-1}$.

B. Oscillations in the ballistic regime

Previously, most analytical studies have focused on strong momentum scattering.^{5,6,7,8,13,14,15,16,17} We will show that the ballistic and weak momentum scattering regimes are far more complex.^{37,38,39,40} In the ballistic limit $\tau_p \rightarrow \infty$ and Eq. (3) can be solved exactly as $\bar{\rho}(t) = \overline{e^{-iHt'/\hbar} \rho_{0\perp} e^{iHt'/\hbar}} + \bar{\rho}_{0\parallel}$, which can also be obtained from the quantum Liouville equation.²⁹ This determines the time evolution of an initial spin polarization $\mathbf{s}(t=0) = \mathbf{s}_0$, i.e., the component of $\mathbf{s}(t)$ along \mathbf{s}_0 . For simplicity \mathbf{s}_0 is here assumed independent of \mathbf{k} ; a \mathbf{k} -dependent distribution would not change the results qualitatively. From the solution for $\bar{\rho}$ in the ballistic limit we have

$$\mathbf{s}(t) \cdot \hat{\mathbf{s}}_0 = [1 - (\hat{\boldsymbol{\Omega}} \cdot \hat{\mathbf{s}}_0)^2] \cos \Omega t + (\hat{\boldsymbol{\Omega}} \cdot \hat{\mathbf{s}}_0)^2, \quad (4)$$

where $\hat{\mathbf{a}}$ denotes the unit vector in the direction of \mathbf{a} . The last term corresponds to $\bar{\rho}_{0\parallel}$. It is best to take a concrete example, such as the Hamiltonian of a 2D system on a (001) surface with linear Rashba⁴¹ and Dresselhaus⁴² spin-orbit interactions, $H = \alpha(\sigma_x k_y - \sigma_y k_x) + \beta(\sigma_x k_x - \sigma_y k_y)$. We consider first effective fields $\boldsymbol{\Omega}(\mathbf{k})$ such that the magnitude $|\boldsymbol{\Omega}(\mathbf{k})|$ does not depend on the direction of \mathbf{k} , for example either $\alpha = 0$ or $\beta = 0$ yields $|\boldsymbol{\Omega}(\mathbf{k})| = \Omega(k)$. In this case the initial spin polarization will simply oscillate with frequency Ω . It is helpful to visualize a population of spins on the Fermi surface, all initially pointing up. If $|\boldsymbol{\Omega}(\mathbf{k})|$ is the same at all points \mathbf{k} on the Fermi surface, all spins that were in phase initially will be in phase again after one precession period. Some fraction of the initially oriented spins \mathbf{s}_0 , corresponding to the last term in Eq. (4), has a nonzero overlap with the local field $\boldsymbol{\Omega}(\mathbf{k})$ so that the projection of \mathbf{s}_0 on $\boldsymbol{\Omega}(\mathbf{k})$ will be preserved. This fraction is zero if the initial spin is out of the plane, but significant if it is in the plane.

C. Non-exponential decay in the ballistic regime

The case when $|\boldsymbol{\Omega}(\mathbf{k})|$ depends on the direction of \mathbf{k} is of great relevance to experiment, where spin-orbit coupling is rarely attributable to a single mechanism. Spins on the Fermi surface precess with incommensurable frequencies and once they are out of phase they never all get in phase again (but the polarization fraction due to $\rho_{0\parallel}$ is conserved.) In our example, if $\beta \ll \alpha$ we can write $\Omega \approx \bar{\Omega} (1 + \eta \sin 2\phi)$, where $\bar{\Omega} \equiv k \sqrt{\alpha^2 + \beta^2}$, the small parameter $\eta \equiv \alpha\beta/[2(\alpha^2 + \beta^2)]$, and ϕ is the polar angle of \mathbf{k} . The motion of the spins, given by Eq.

(4), is averaged over the Fermi circle. Consider the term $\cos \Omega t$ in Eq. (4) as an example. The angular average yields $\cos(\Omega t) J_0(\eta \bar{\Omega} t)$, where J_0 is a Bessel function of the first kind. This function has the form of a decaying oscillation but it does *not* reduce to an exponentially damped oscillation in any limit. At long times we have $J_0(\eta \bar{\Omega} t) \rightarrow \sqrt{2/(\pi \eta \bar{\Omega} t)} \cos(\eta \bar{\Omega} t - \pi/4)$. A similar, more complicated expression in terms of Bessel functions applies for the remaining term $(\hat{\Omega} \cdot \hat{s}_0)^2 \cos \Omega t$ in Eq. (4).

The anisotropy of the Fermi surface introduces a mechanism for non-exponential spin decay⁴³ with a characteristic time $\tau_d \propto (\eta \bar{\Omega})^{-1}$, referred to as the dephasing time τ_d . For pure spin dephasing, i.e., in the absence of momentum scattering, two limiting cases can be distinguished. If $\Omega(\mathbf{k}) \perp \mathbf{s}_0$ for all \mathbf{k} (e.g., a spin orientation perpendicular to the 2D plane on a [001] surface), spin dephasing reduces the spin polarization to zero. On the other hand, spin dephasing is completely suppressed if $\Omega(\mathbf{k}) \parallel \mathbf{s}_0$ for all \mathbf{k} (e.g., a 2D electron system in a symmetric quantum well on a [110] surface with a spin orientation perpendicular to the 2D plane⁶). In general (in particular for 3D systems), an intermediate situation is realized where the spin polarization is reduced because of dephasing, but it remains finite. The surviving part is identified with $\rho_{0\parallel}$ in the initial density matrix. This process is referred to as *incomplete* spin dephasing.

Analogous results hold for the k^3 -Dresselhaus model,⁴² but the terms leading to dephasing cannot be expressed in a simple form due to the complex angular dependence of $|\Omega(\mathbf{k})|$ on the direction of \mathbf{k} . Figure 1(a) shows the incomplete dephasing of electron spins in bulk GaAs calculated using the k^3 -Dresselhaus model. At long times the initial spin polarization settles to a value ≈ 0.33 , which is independent of any system parameters, including the spin-orbit constant.

D. Weak momentum scattering regime

In the regime of weak momentum scattering the solution to Eq. (3) may be written approximately as

$$\bar{\rho}(t) = \bar{\rho}_{0\parallel} + e^{-t/\tau_p} \overline{e^{-iHt'/\hbar} \rho_{0\perp} e^{iHt'/\hbar}}. \quad (5)$$

Since the momentum scattering rate $1/\tau_p$ is small, the term under the overline is taken to lowest order in $1/\tau_p$. The second term on the RHS of Eq. (5) describes damped oscillations with amplitude decaying exponentially on a scale $\propto \tau_p$. This trend is the *inverse* of that for strong momentum scattering and is explained by the following argument. If one spin, precessing on the Fermi surface in phase with all the other spins, is scattered to a different wave vector, it will precess about a different effective field and will no longer be in phase with the other spins. Thus the combined effect of spin precession and momentum scattering—even when the latter is only weak—reduces the spin polarization faster. The fraction of the spin polarization corresponding to $\rho_{0\parallel}$ survives. This remaining

polarization decays via spin-flip scattering (the Elliott-Yafet mechanism^{9,10}) on much longer time scales.

An exception occurs when $\Omega(\mathbf{k}) \parallel \mathbf{s}_0$ for *all* spins. This situation is realized, e.g., for a 2D electron system in a symmetric quantum well on a [110] surface with a spin orientation perpendicular to the 2D plane. For this particular case it is well known that spin precession and momentum scattering do not affect at all the initial spin orientation.^{6,22} From the preceding discussion we can understand this by noting that the initial density matrix $\rho_0 = \rho_{0\parallel}$ commutes with the spin-orbit Hamiltonian and Eq. (3) shows that $\bar{\rho} = \bar{\rho}_{0\parallel}$ for all times. The polarization decays eventually via spin-flip scattering.^{9,10}

In the weak momentum scattering regime for non-negligible anisotropy the spin decay rate is determined by the larger of $\eta \bar{\Omega}$ and $1/\tau_p$. Momentum scattering introduces an exponential envelope but in this limit the concept of a spin relaxation time is evidently of limited use.⁴⁴ In spin-1/2 systems dephasing will be important for high-mobility carriers. Results consistent with our findings were obtained experimentally by Brand *et al.*⁴⁵ who studied the oscillatory time evolution of an optically-generated spin polarization in a high-mobility 2D electron system in a GaAs/AlGaAs quantum well. Similarly, in materials in which a nonequilibrium spin density is excited, the time evolution of this spin density can be studied, for example, by means of magnetic circular dichroism techniques.⁴⁶

We have assumed an initial spin distribution sharp at the Fermi edge. In practice this distribution spans a window in \mathbf{k} -space, introducing additional dephasing between spins at wave vectors of slightly different magnitudes. In this case even an isotropic spin splitting leads to decay, though the polarization due to $\rho_{0\parallel}$ is still robust. For example, in 2D for isotropic spin splitting, $\int dk k \cos \Omega t \propto t^{-2}$, so that the spin polarization, instead of oscillating indefinitely, decays as t^{-2} .

We have also assumed the initial spin distribution to be independent of wave vector. The theory is well-equipped to deal with wave-vector dependent spin polarizations. (Indeed, the initial spin distribution, contained in the density matrix ρ_0 , is in general wave vector-dependent.) The wave vector-independent cases discussed at length are intended as examples, and they have been selected as more straightforward cases for clarity.

IV. SPIN-3/2 HOLE SYSTEMS

Next we discuss spin-3/2 hole systems, which are different from spin-1/2 electron systems for several reasons. The presence of extra terms in the spin density matrix of spin-3/2 systems (in addition to the number density and spin polarization) has important consequences for spin dynamics.^{29,47} Spin-orbit coupling affects the energy spectrum in the valence band to a greater extent, and the spin orientation often disappears on scales comparable to τ_p . The relation $\Omega \tau_p \ll 1$ holds less frequently than for

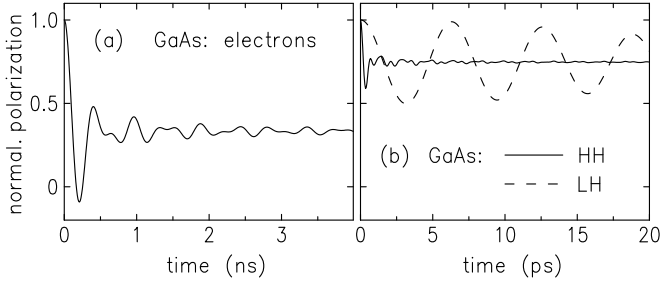


FIG. 1: Incomplete spin dephasing of (a) electron spins in the k^3 -Dresselhaus model and (b) of heavy-hole spins (solid lines) and light-hole spins (dashed lines) in bulk GaAs in the ballistic limit. The vertical axis shows the normalized spin polarization $\mathbf{s}(t) \cdot \hat{\mathbf{s}}_0 / |\mathbf{s}_0|$. The initial spin polarization \mathbf{s}_0 is assumed to point along [001]. The Fermi wave vector is $k_F = 10^8 \text{ m}^{-1}$. Note the different time scales in (a) and (b).

electrons in systems accessible experimentally.

We consider spin-3/2 holes to be described by the Luttinger Hamiltonian,^{48,49}

$$H_0 = \frac{\hbar^2}{2m_0} [(\gamma_1 + \frac{5}{2}\bar{\gamma})k^2 - 2\bar{\gamma}(\mathbf{k} \cdot \mathbf{S})^2] - H_C, \quad (6)$$

where m_0 is the bare electron mass, \mathbf{S} the spin operator for effective spin 3/2, $\bar{\gamma} \equiv (2\gamma_2 + 3\gamma_3)/5$, and γ_1 , γ_2 , and γ_3 are Luttinger parameters. H_C represents the anisotropic terms with cubic symmetry⁴⁹ which will be given below.

We work first in the spherical approximation in which H_C is neglected. The energy dispersions E_{HH} for the heavy holes (HHs, spin projection in the direction of \mathbf{k} is $m_s = \pm 3/2$) and E_{LH} for the light holes (LHs, $m_s = \pm 1/2$) are $E_{\text{LH/HH}}(\mathbf{k}) = \hbar^2 k^2 (\gamma_1 \pm 2\bar{\gamma}) / (2m_0)$.

A. Exponential decay in the strong momentum scattering regime

In the strong momentum scattering limit an exponential solution $\bar{\rho}(t) = e^{-\Gamma_s t} \bar{\rho}_0$ is possible. The tensor $\Gamma_s = \tau_s^{-1} \mathbb{1}$, showing that the relaxation times for all spin components are equal,

$$\frac{1}{\tau_s} = \frac{2}{5} \Omega^2 \tau_p = \frac{8}{5} \left(\frac{\hbar \bar{\gamma} k^2}{m_0} \right)^2 \tau_p, \quad (7)$$

where now the frequency $\Omega(\mathbf{k}) = (E_{\text{LH}} - E_{\text{HH}})/\hbar = 2\hbar \bar{\gamma} k^2 / m_0$ corresponds to the energy difference between the HH and LH bands.²⁹ Despite the qualitatively different spin precession, the situation is rather similar to electron spin relaxation and can be explained in terms of the same random walk picture familiar from the study of electron spin relaxation.

B. Ballistic and weak momentum scattering regimes in the spherical approximation

In the ballistic limit Eq. (3) is again solved by $\bar{\rho}(t) = e^{-iHt'/\hbar} \bar{\rho}_0 e^{iHt'/\hbar}$. An initial spin polarization will oscillate indefinitely since Ω is the same for all holes on the Fermi surface. For weak momentum scattering Eq. (5) applies to holes also. The spin polarization consists of damped oscillations, decaying on a time scale $\propto \tau_p$, plus a term corresponding to $\rho_{0\parallel}$, which survives at long times. $\rho_{0\parallel}$ does not depend on the Luttinger parameters or the Fermi wave vector and will therefore be the same in any system described by the Luttinger Hamiltonian. This remaining polarization decays via spin-flip scattering as discussed in Refs. 34,35.

C. Cubic-symmetry terms and dephasing

Dephasing is introduced if the term H_C with cubic symmetry⁴⁹ is included in the Luttinger Hamiltonian,

$$H_C = \frac{\hbar^2 \Delta}{m_0} (k_x k_y \{J_x, J_y\} + k_y k_z \{J_y, J_z\} + k_z k_x \{J_z, J_x\}), \quad (8)$$

where $\Delta = (\gamma_3 - \gamma_2)/2$. The eigenenergies are now given by $E_{\text{HH/LH}} = \gamma_1 \hbar^2 k^2 / 2m_0 \pm 2\sqrt{E_{\text{an}}^2}$, where

$$E_{\text{an}}^2 = \bar{\gamma} k^4 - 2\bar{\gamma} \Delta [k^4 - 6(k_x^2 k_y^2 + k_y^2 k_z^2 + k_z^2 k_x^2)] + k^4 \Delta^2. \quad (9)$$

The cubic-symmetry terms contained in Eq. (8) are usually neglected in charge and spin transport without a significant loss of accuracy. However, they play a crucial role in spin relaxation in the weak momentum scattering regime, which for holes extends over a wide range of k .

Due to the presence of H_C , the energy dispersion relations and therefore $|\Omega(\mathbf{k})|$ depend on the direction of \mathbf{k} , causing an initial spin polarization to decay even in the ballistic limit, where incomplete spin dephasing occurs. Our numerical calculations exemplified in Fig. 1(b) show that an initial spin polarization falls to a fraction much higher than in the electron cases studied. It decays more slowly for the LHs, for which the Fermi surface is nearly spherical, than for the HHs, for which the Fermi surface deviates significantly from a sphere. At long times the initial spin polarization settles to a value ≈ 0.75 , which is independent of any system parameters, including the Luttinger parameters.

V. SUMMARY

In conclusion, we have shown that the decay of spin polarization in semiconductors, brought about by the interplay of spin precession and momentum scattering, depends strongly on the regime of momentum scattering. In the ballistic regime the spin polarization decays via a dephasing mechanism which is present due to the fact

that the magnitude of the spin-orbit interaction generally depends on the direction of the wave vector. This mechanism may reduce a spin polarization to zero (complete dephasing) or a fraction of the initial value (incomplete dephasing). Weak momentum scattering destroys an initial spin polarization, whereas strong momentum scattering helps maintain an initial polarization.

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